Zero-energy states in graphene

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The current interest in zero-energy states in condensed matter systems with linear quasi-particle dispersion is mostly driven by the on-going search for Majorana fermions in topological insulators. We argue that zero-energy states in graphene, which exist independently of the sign of the confinement potential but are not self-adjoint, also provide a diverse range of interesting problems with important practical implications. For example, there is a widespread belief that electrostatic confinement of graphene charge carriers, which resemble massless Dirac fermions, is impossible as a result of the Klein paradox. We show that full confinement is indeed possible for zero-energy states in pristine graphene. We present exact analytical solutions for the zero-energy modes of two-dimensional massless Dirac fermions confined within a smooth one-dimensional potential given by hyperbolic secant [1], which provides a reasonable fit for the potential profiles of existing top-gated graphene structures [2,3]. A simple relationship between the characteristic strength and the number of confined modes within this model potential is found.

A numerical method for finding the number of fully confined zero-energy modes in any smooth potential, decaying at large distances faster than the Coulomb potential, has also been developed and used to evaluate the conductivity of a channel formed by a more realistic top-gate potential [4]. The long-range behavior of the potential defines the threshold condition for confinement, with power-decaying potentials demonstrating the absence of threshold in the potential strength for the appearance of at least one bound state which is different from exponentially-decaying and square well models. An experimental setup is proposed for the observation of confined electronic guided modes.

We also show that full confinement is possible for zero-energy states in electrostatically-defined quantum dots and rings with smooth potential profiles. The necessary condition for confinement is a non-zero value of angular momentum, i.e. the confined states are vortices. Analytic solutions are found for a class of model potentials [5]. These exact solutions allow us to draw conclusions on general requirements for the potential to support fully confined states, including a critical value of the potential strength and spatial extent. The implications of fully-confined zero-energy states for STM measurements and minimal conductivity are discussed.

We demonstrate that the excitonic insulator gap predicted some time ago [6] and revisited recently by several theory groups [7,8] cannot exist in back-gated graphene samples as confirmed by experiments [9]. A qualitatively different picture based on Bose-Einstein condensation of zero-energy electron-hole vortices (excitons) is proposed to explain the Fermi velocity renormalization in gated graphene structures which is observed instead of the gap.

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