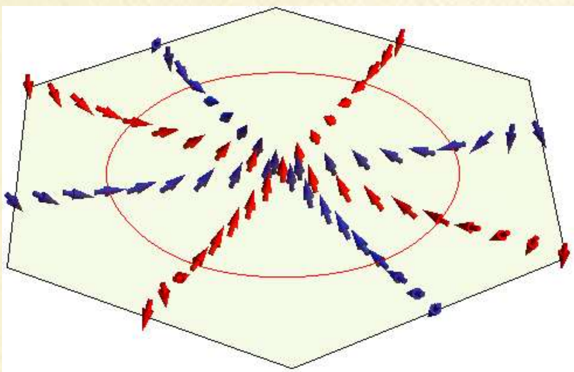
Density Waves Instability and a Skyrmion Lattice on the Surface of Strong Topological Insulators



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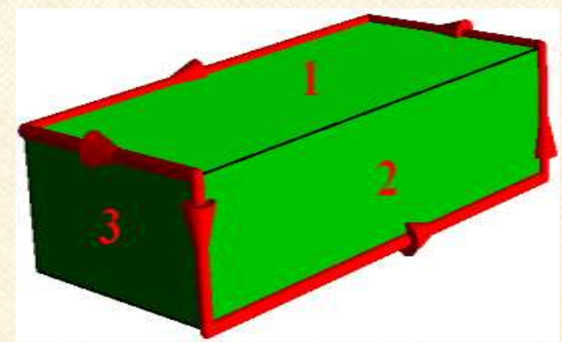
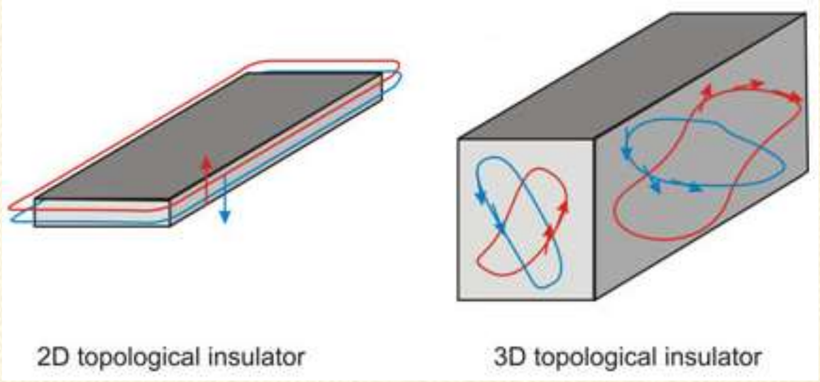


- Phys. Rev. B 86, 195116 (2012)
- Phys. Rev. B 85, 121105(R) (2012)

EP2DS-20, Wroclaw- July 2013

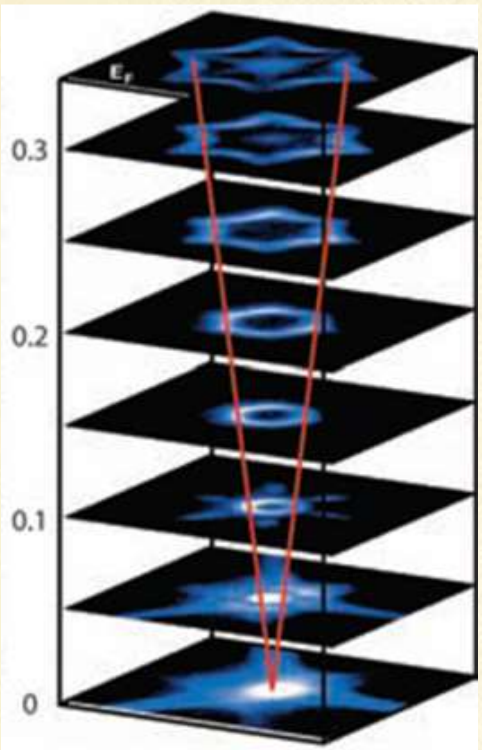
Topological Insulators

3D materials that have a bulk gap like an ordinary insulator, but have conducting states on their surface.



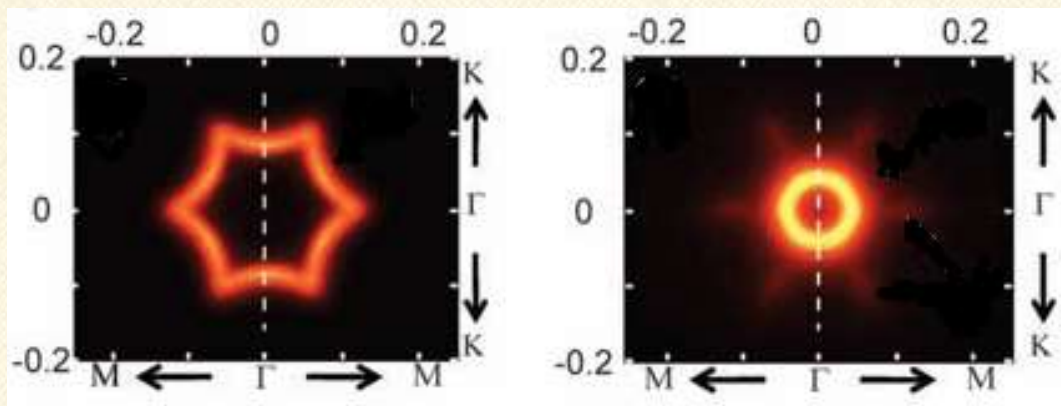
The surface states are protected as long as time-reversal symmetry is maintained.

Surface States - Measurement



Bi_2Te_3

Hexagonal warping of the Fermi surface occurs away from the Dirac point.



Fisher *et al.*
Ando *et al.*

Surface States – Fu Model

2D effective model for the surface states:

$$H_0 = v_0\left(k_x\sigma_y - k_y\sigma_x\right) + \lambda k^3 \cos 3\theta \cdot \sigma_z$$

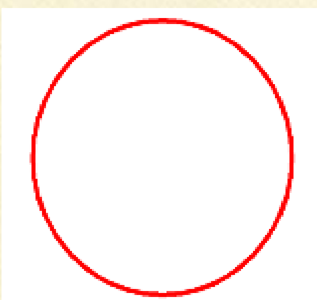
Energy scale:

$$E^* = \sqrt{\frac{v_0^{\,3}}{\lambda}}$$

Surface States

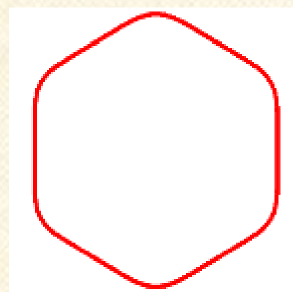
The Fermi-surface can be classified to three regions:

$E_F/E^* < 0.55$



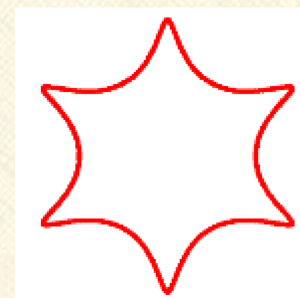
Circular

$0.55 < E_F/E^* < 0.9$



Hexagonal

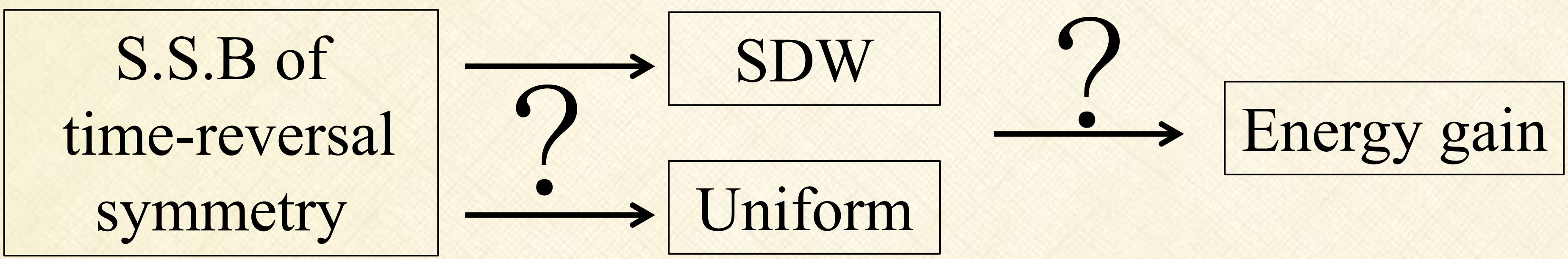
$0.9 < E_F/E^*$



‘Snowflake’

Question

May the surface of a TI lower its energy by a spontaneous breaking of time–reversal symmetry through the formation of a spin polarization ?



The Landau Free-Energy

The Landau free-energy up to a second order:

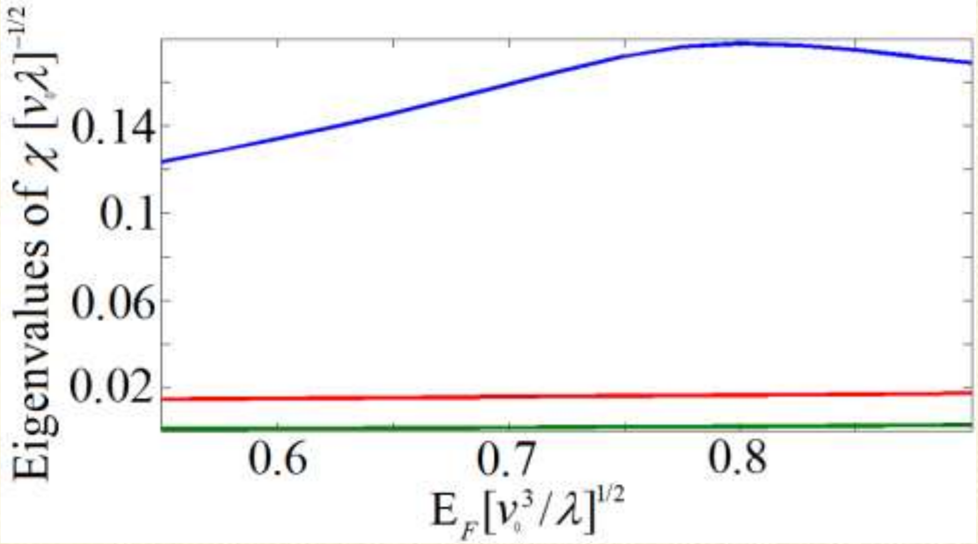
$$F_L = \sum_q m_{-q}^{\mu} \chi_q^{\ \mu\nu} m_q^{\nu} = \sum_q \chi_q \left| \tilde{m}_q \right|^2$$

- Negative eigenvalues = SSB.

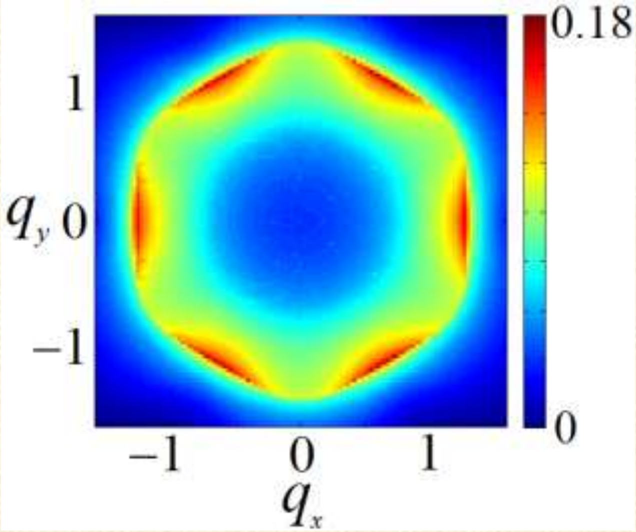
The Spin Susceptibility (non-interacting)

The eigenvalues of the susceptibility matrix:

$\boldsymbol{q} = (2k_F, 0),\ T = 0$



$E_F = 0.7E^*,\ T = 0$



Hexagonal range = Strong nesting = Large susceptibility

The Interacting Spin Susceptibility

Contact interaction:

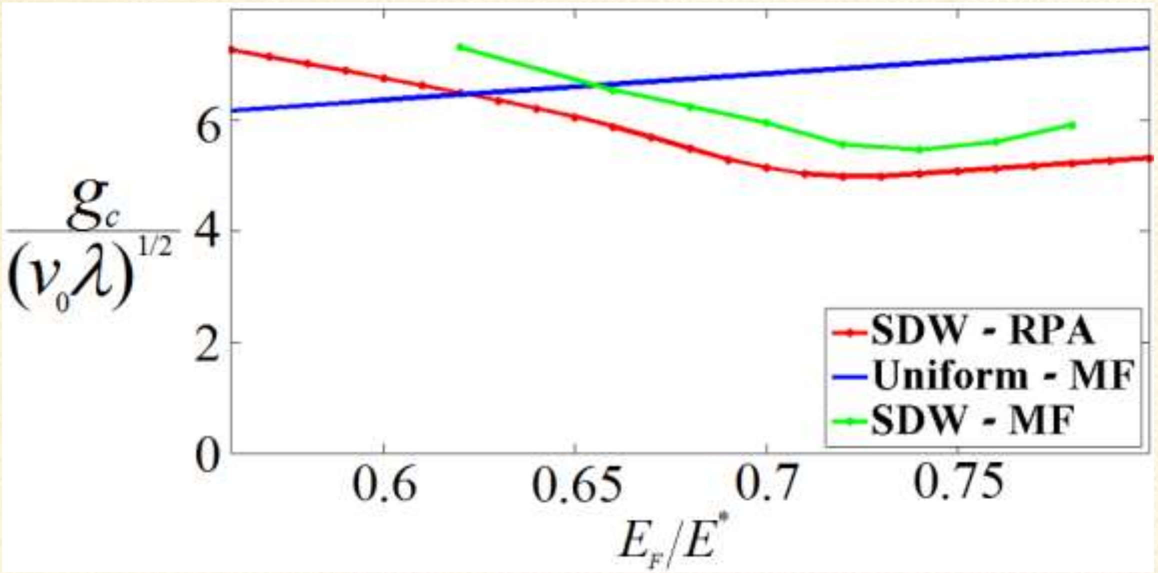
$$V(r-r')=g\delta(r-r') \;\rightarrow\; V_q=g$$

The Random Phase Approximation (RPA):

$$\chi^{\mu\nu}={\chi_0}^{\mu\rho}\left[\left(1-g\chi_0\right)^{-1}\right]^{\rho\mu}\rightarrow\chi=\frac{\chi_0}{1-g\chi_0}$$

$$g_c=\frac{1}{\max(\chi_0)}$$

The Critical Interaction (T=0)



- Circular region → no nesting → the uniform state is favored.
- Hexagonal region → nesting → the SDW state is favored.

The Magnetization

Each nesting vectors **q**

maximal eigenvalue χ_q ⟷ normalized eigenvector

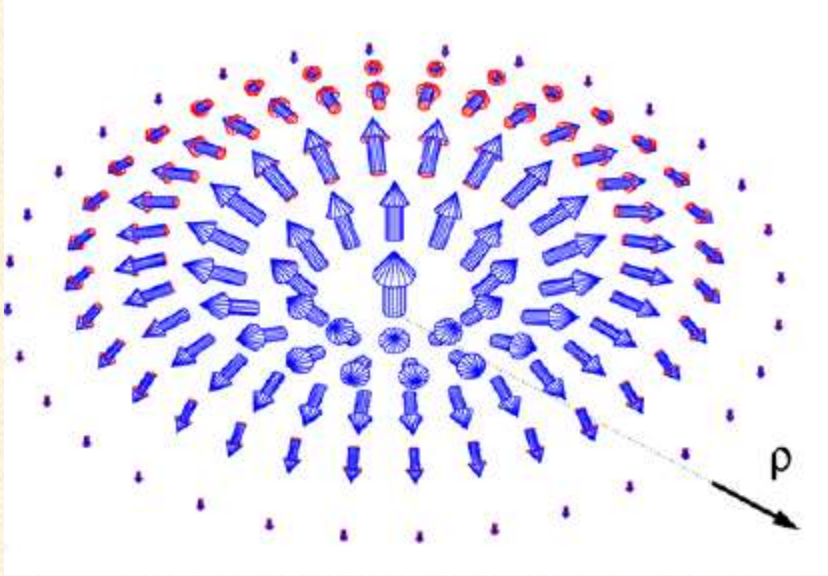
$$\hat{n}_q \equiv \frac{1}{\sqrt{2}}\left(\hat{V}_q + i\hat{U}_q\right)$$

$$\boldsymbol{m(r)} \propto \sum_{\boldsymbol{q}} \hat{V}_q \cos(\boldsymbol{q}\cdot\boldsymbol{r}+\varphi_q) + \hat{U}_q \sin(\boldsymbol{q}\cdot\boldsymbol{r}+\varphi_q)$$

- Each nesting vector **q** contributes to the magnetization a spiral SDW propagating in the **q** direction.

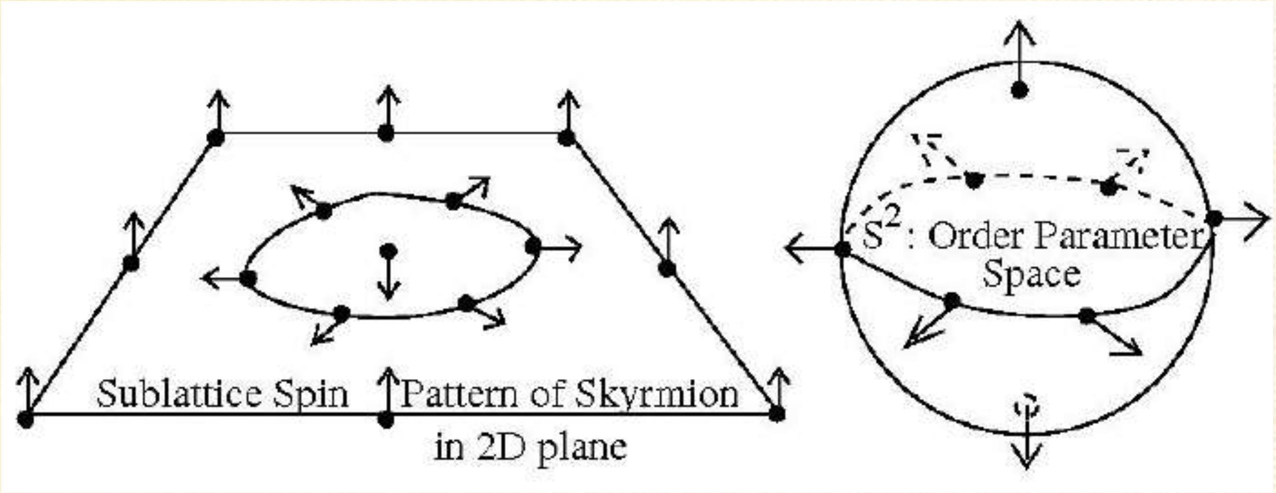
Skyrmion Lattice

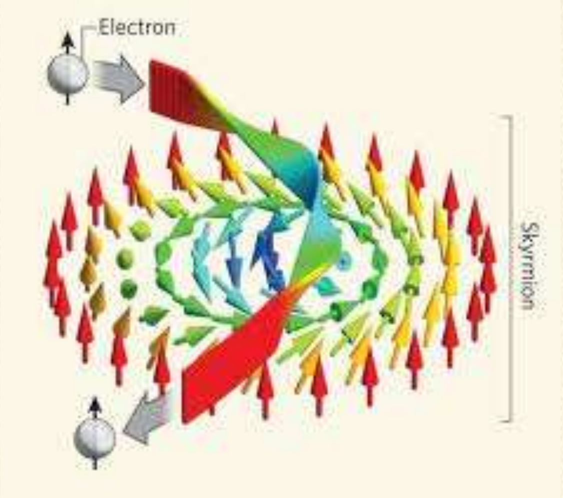
Superposition of the spin-spirals → Skyrmion-lattice.



Skyrmion

Skyrmion = topologically non trivial (stable) spin texture.





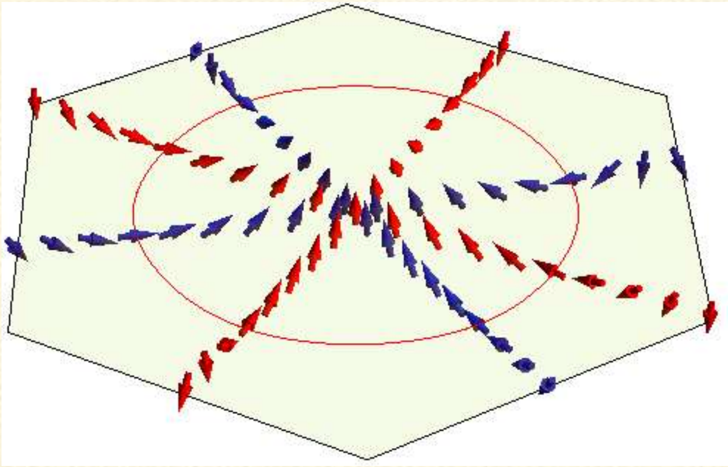
$$Q_{top} = \frac{1}{4\pi} \iint dxdy \left[\hat{m} \cdot (\partial_x \hat{m} \times \partial_y \hat{m}) \right]$$

Skyrmion Lattice

Superposition of the spin-spirals→triangular Skyrmion-lattice:

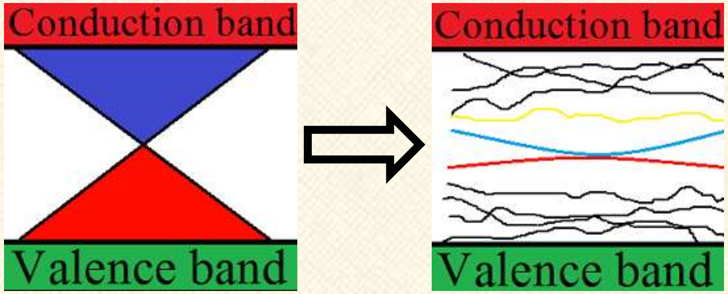
$m_i(\boldsymbol{r})$ form a periodic structure with a triangular symmetry.

Each unit cell holds a Skyrmion with $Q_{top} = \pm 1$.



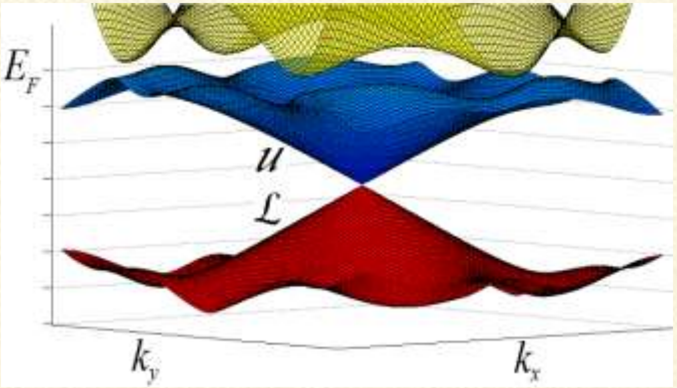
Topological properties

- From continuum “Dirac-cone” to a periodic band structure.



- The Fermi-energy always lies in a gap.

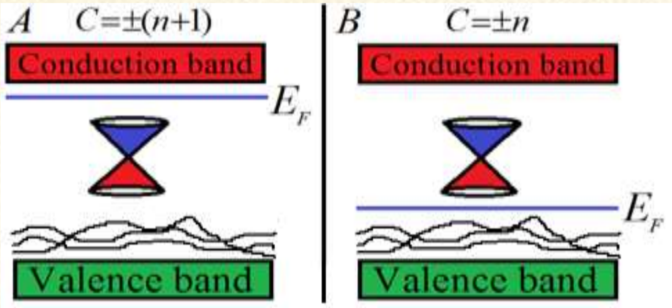
- Non-trivial Chern-number arises in the absence of any external field.



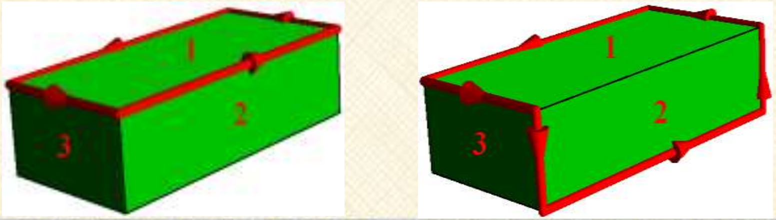
$$C_{L+U} = Q_{top} = \pm 1$$

Topological properties

- Tuning E_F ⇨ Chern-number is changed by an odd integer.



- Tuning E_F ⇨ Network of 1D chiral channels.



Summary

1. For a strong enough interaction the surface of TI may be unstable to the formation of a Skyrmion-lattice.
2. Even in the absence of an external magnetic field a non trivial Chern-number arises.
3. A network of one dimensional chiral channels may be established on the surfaces of a topological insulator.