BROKEN TRANSLATION SYMMETRY AND EDGE STATES

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PURPOSE

Illustration of the interplay of Edge State and Translation

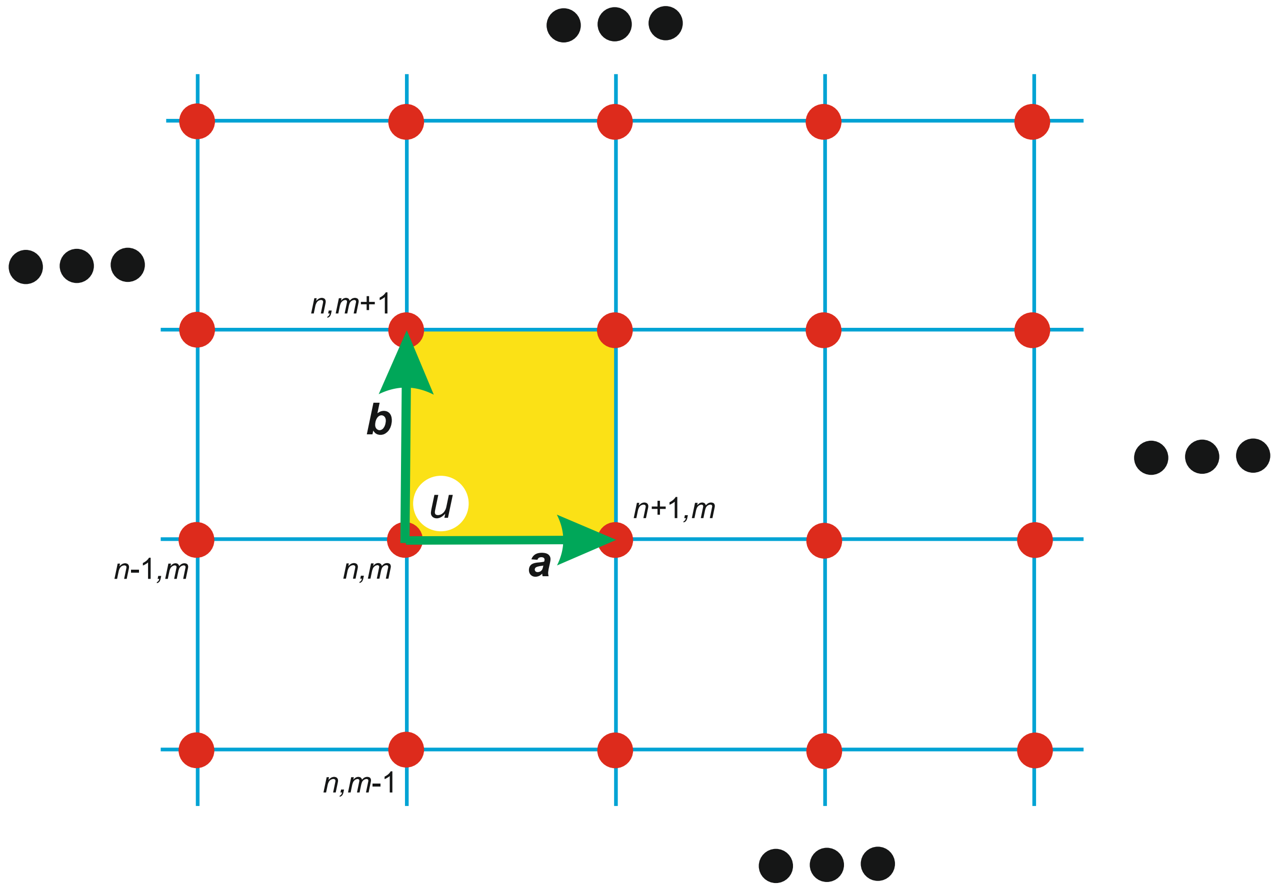
PROBLEMS:

$2D \longrightarrow 1D$

Bethe Ansatz for 1D

Properties

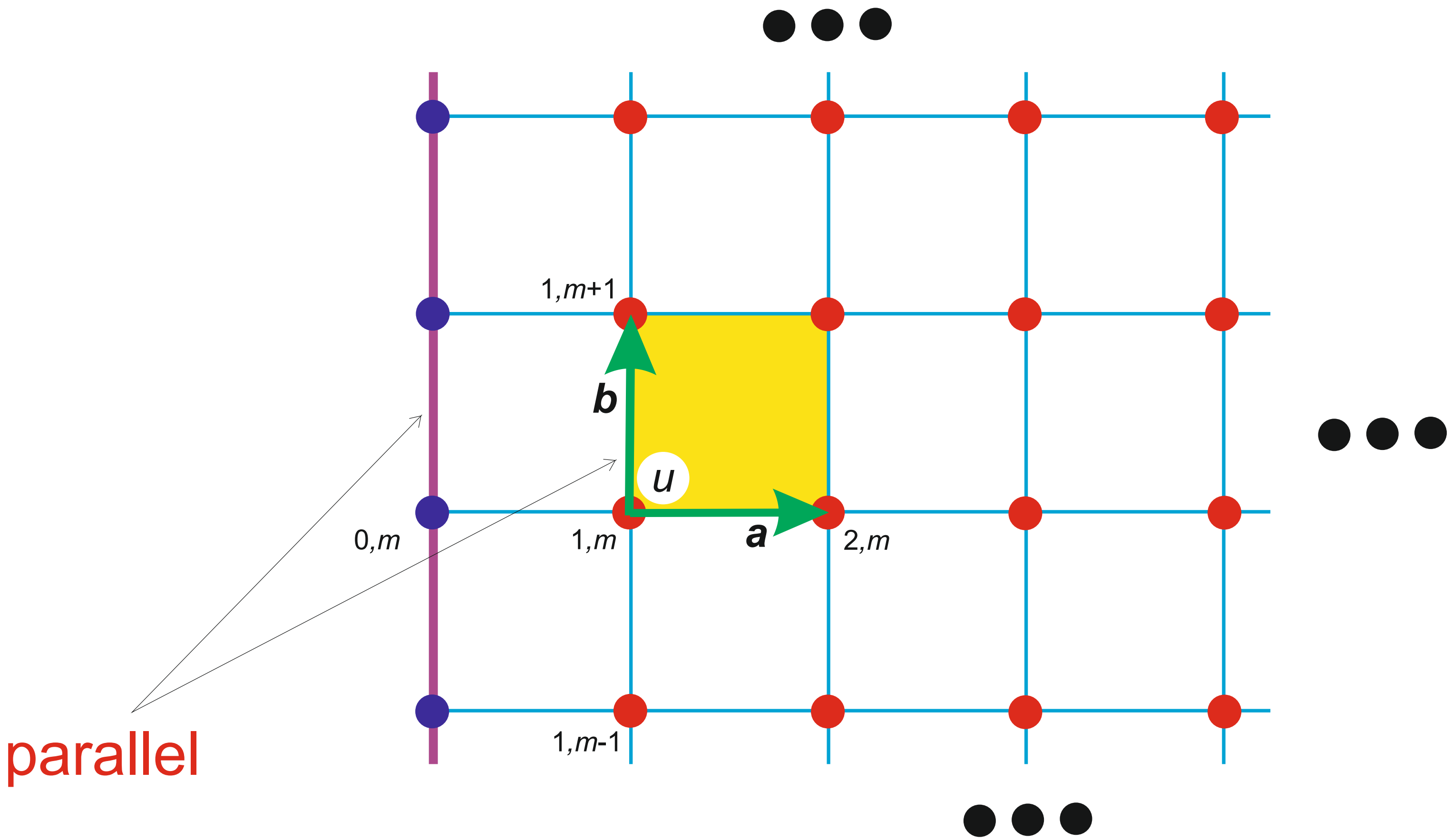
MODEL



TBM:

$$E u_{n,m} = -\left(u_{n+1,m} + u_{n-1,m} + u_{n,m+1} + u_{n,m-1}\right)$$

MOST SYMMETRIC EDGE



$$Eu_{n,m} = -\left(u_{n+1,m} + u_{n-1,m} + u_{n,m+1} + u_{n,m-1}\right), \quad n \geqslant 1$$

Boundary condition: $(E - U)u_{0,m} = -u_{1,m} - s\left(u_{0,m+1} + u_{0,m-1}\right)$

2D → 1D

Translation operator along the edge $Tu_{n,m} = u_{n,m+1}$

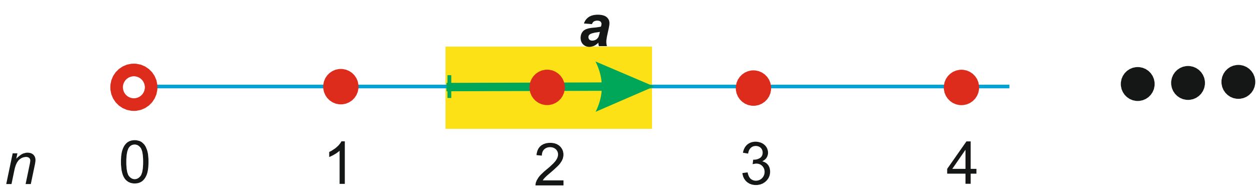
It's eigenfunction $\exp(iqm)$

Solution of 2D lattice problem $u_{n,m} = \exp(iqm)u_n$

Effective 1D chain problem

$$\left[E - U^{\text{eff}}(q)\right] u_n = -\left(u_{n+1} + u_{n-1}\right), \quad n \geqslant 1; \quad U^{\text{eff}}(q) = -2\cos q$$

$$\left[E - U^{\text{eff}}(q) - U_0(q)\right] u_0 = -u_1, \quad U_0(q) = U + (1-s)U^{\text{eff}}(q)$$



BETHE ANSATZ

1D edge state probem

Eq.: $$Eu_n = -(u_{n+1} + u_{n-1}), \quad n \geqslant 1$$

B.c.: $$(E - U)\, u_0 = -u_1$$

Eq. can be satisfied with $u_n = \exp(\mathrm{i}kn)$

what leads to spectrum $E = -2\cos k$

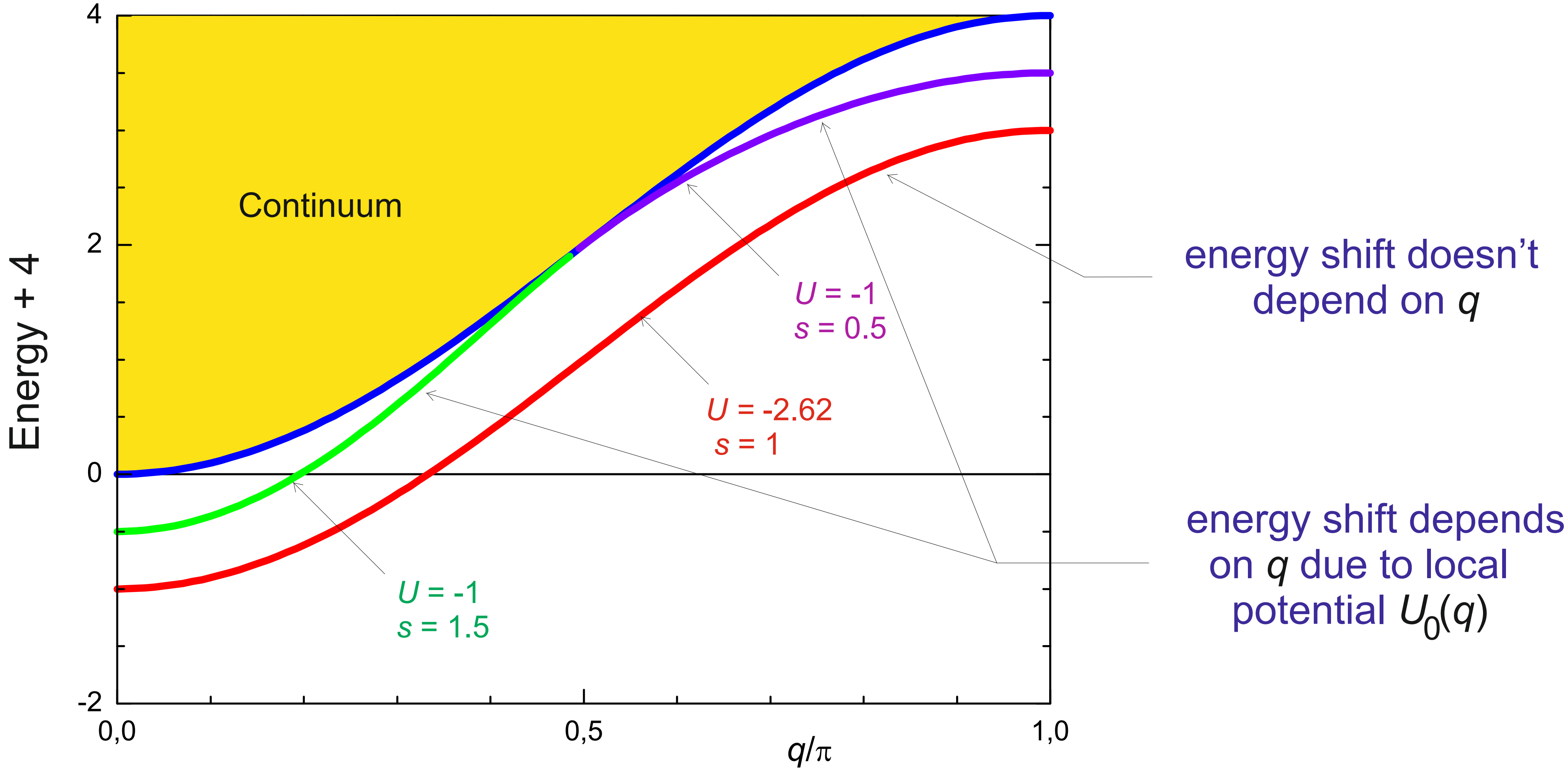
Edge state: $k \to \mathrm{i}\kappa \quad \longrightarrow \quad u_n = \mathrm{e}^{-\kappa n}$

Satisfying B.c. we obtain $U = -\mathrm{e}^{\kappa}$ and edge state energy

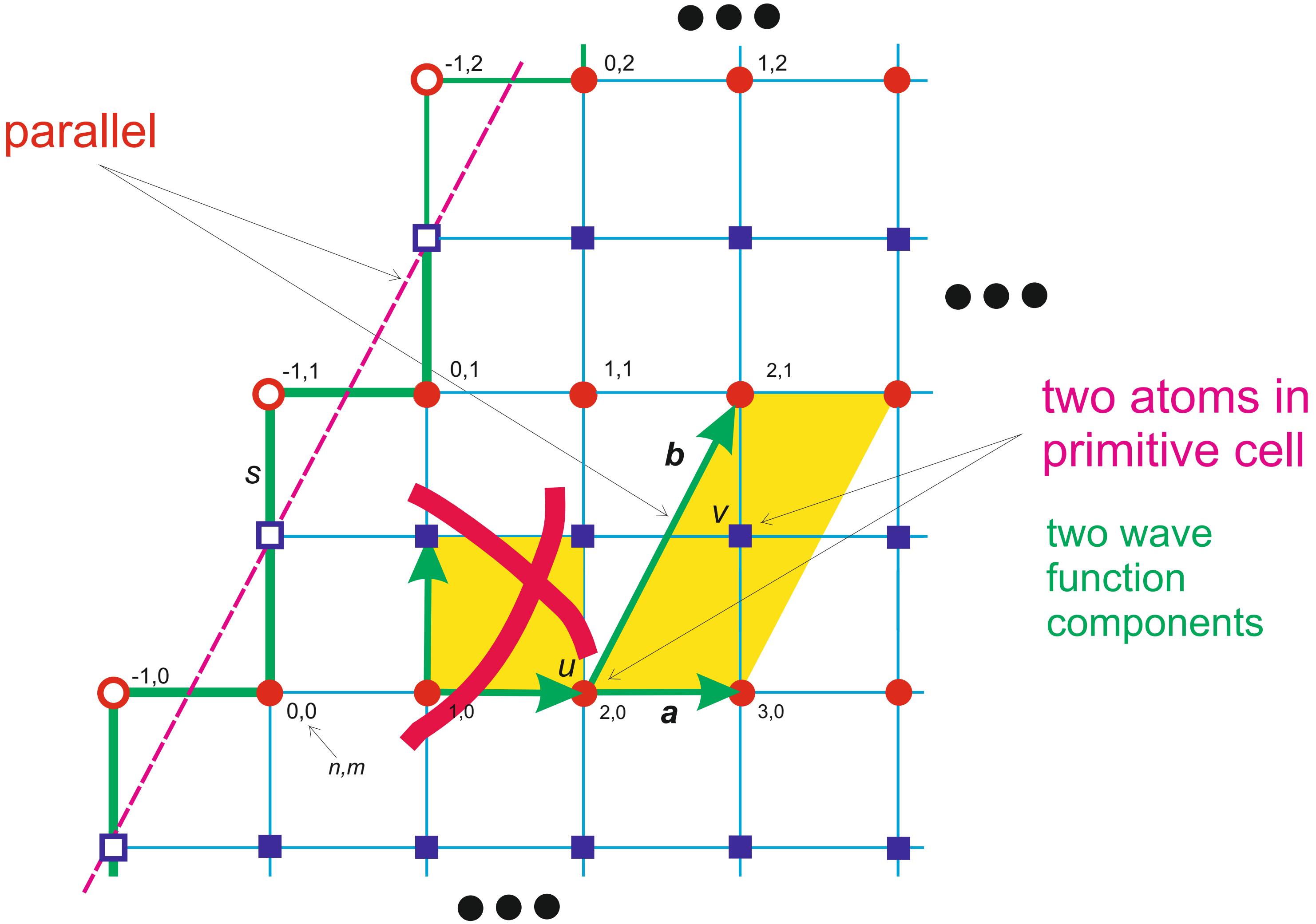
$$E = U + 1/U \quad \text{if} \quad U < -1$$

SPECTRUM

in the case of the most symmetric edge



TILTED EDGE



parallel
two atoms in primitive cell
two wave function components
-1,2
0,2
1,2
-1,1
0,1
1,1
2,1
-1,0
0,0
1,0
2,0
3,0
n,m
s
b
v
u
a

2D → 1D

Translation along the edge suggests the following

solution of 2D lattice problem

$$\begin{pmatrix} u_{n,m} \\ v_{n,m} \end{pmatrix} = \mathrm{e}^{\mathrm{i}qm} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

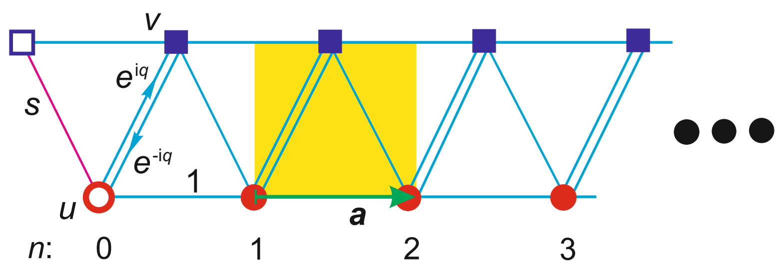
Eqs.

$$\begin{cases} Eu_n = -\left(u_{n-1} + u_{n+1} + \mathrm{e}^{-\mathrm{i}q} v_n + v_{n-1}\right), & n \geqslant 1; \\ Ev_n = -\left(v_{n-1} + v_{n+1} + u_{n+1} + \mathrm{e}^{\mathrm{i}q} u_n\right), & n \geqslant 0; \end{cases}$$

$$\left.\begin{aligned} (E-U)u_0 &= -\left(u_1 + \mathrm{e}^{-\mathrm{i}q} v_0 + s v_{-1}\right), \\ (E-V)v_{-1} &= -\left(v_0 + s u_0\right) \end{aligned}\right\}$$

B.cond.

Effective 1D chain problem

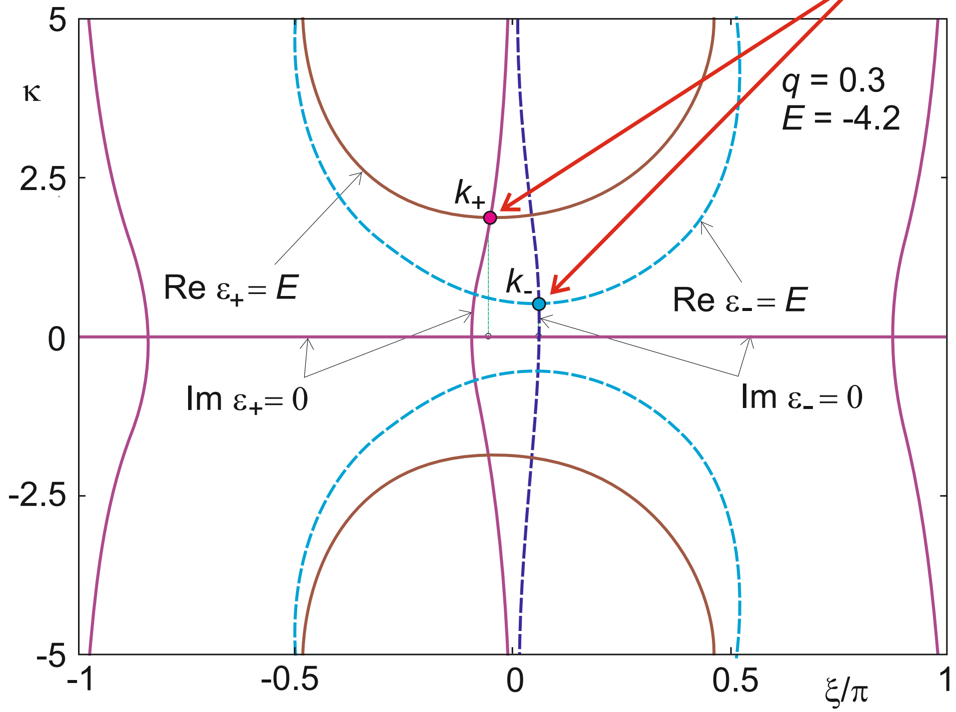


BETHE ANSATZ

Eqs. can be satisfied with $\psi_n = \begin{pmatrix} u_n \\ v_n \end{pmatrix} = \mathrm{e}^{\mathrm{i}kn} \begin{pmatrix} u \\ v \end{pmatrix}$

what leads to spectrum $E_\pm = -2\cos k \pm 2\cos\{(k-q)/2\}$

Two momenta $k \to \xi + \mathrm{i}\kappa, \quad \kappa > 0$



in the case of given energy E<-4.

Using the superposition

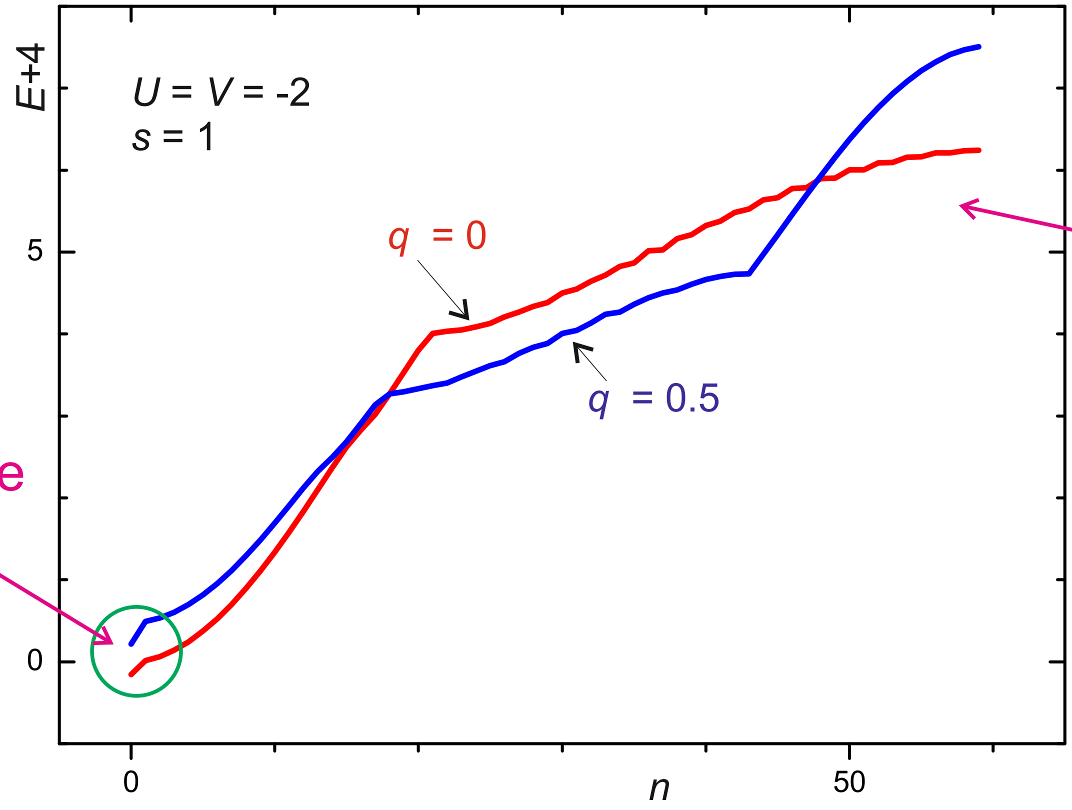
$$\begin{pmatrix} u_+ \\ v_+ \end{pmatrix} \mathrm{e}^{\mathrm{i}k_+ n} + A \begin{pmatrix} u_- \\ v_- \end{pmatrix} \mathrm{e}^{\mathrm{i}k_- n}$$

we have to satisfy 4 real B.cond equations with only 3 parameters: Re A, Im A and E.

We managed to do that, proving that the Bethe Ansatz works in this tilted edge case.

NUMERICAL CHECK

1D chain equations in matrix format: $$(H - E)\begin{pmatrix} u \\ v \end{pmatrix} = 0$$



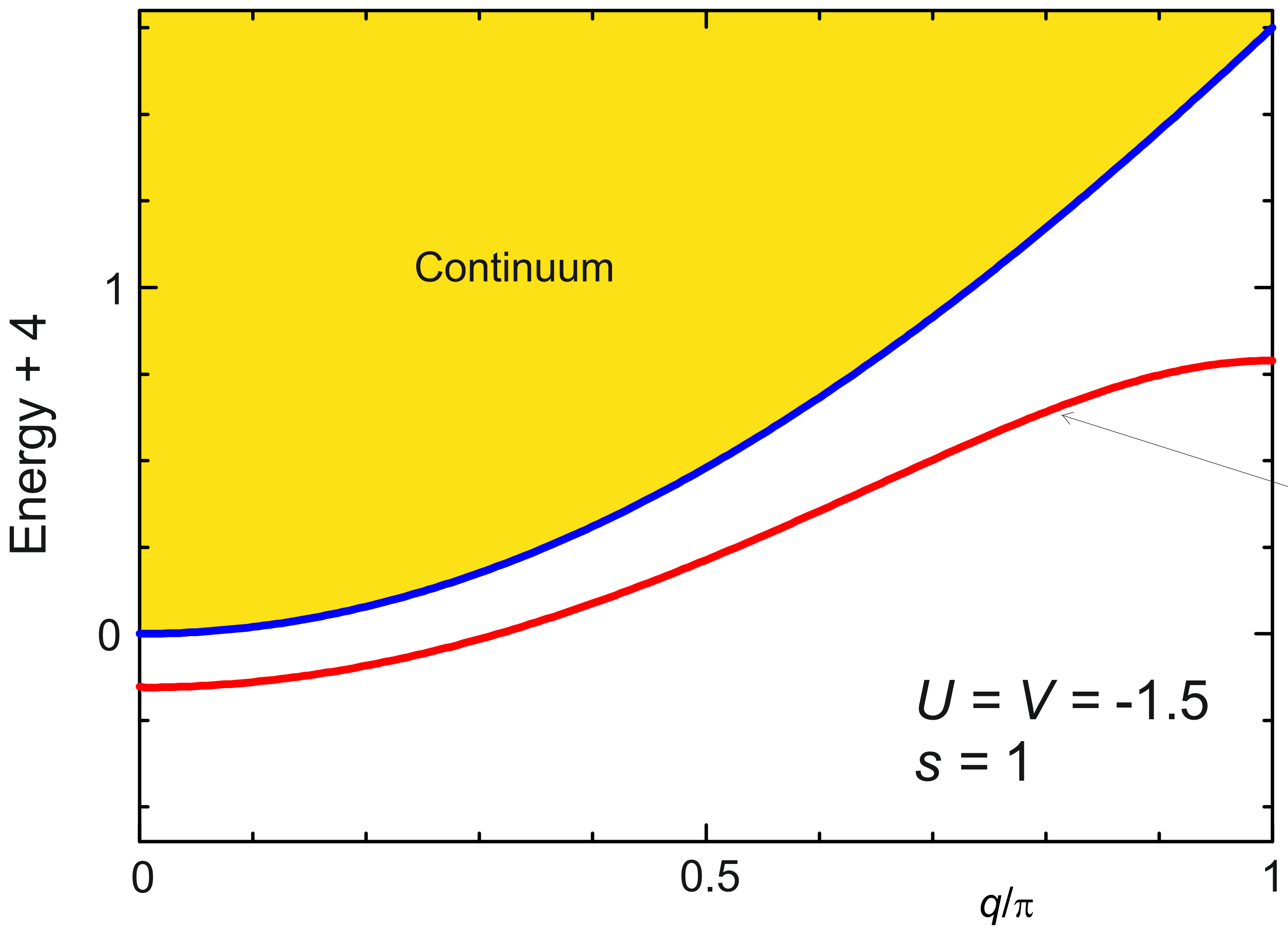
Edge state

H diagonalization for the ribbon of 30 primitive cells

Edge state energy coincides with the one obtained by Bethe Ansatz method within 6 digit accuracy

SPECTRUM

in the case of the tilted edge



energy shift depends on *q* due to the coupling of parallel and perpen-dicular motions

CONCLUSIONS

- Edge breaks the translation symmetry
- Enlarging the primitive cell one can restore the translation symmetry along the edge
- That enables to reduce the 2D problem to 1D one, although by enlarging the number of wave function components
- The 1D problem can be solved by Bethe Ansatz, because the edge breaks the translation symmetry only locally

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