Spin-orbit coupling and the pining of stripes in high Landau levels

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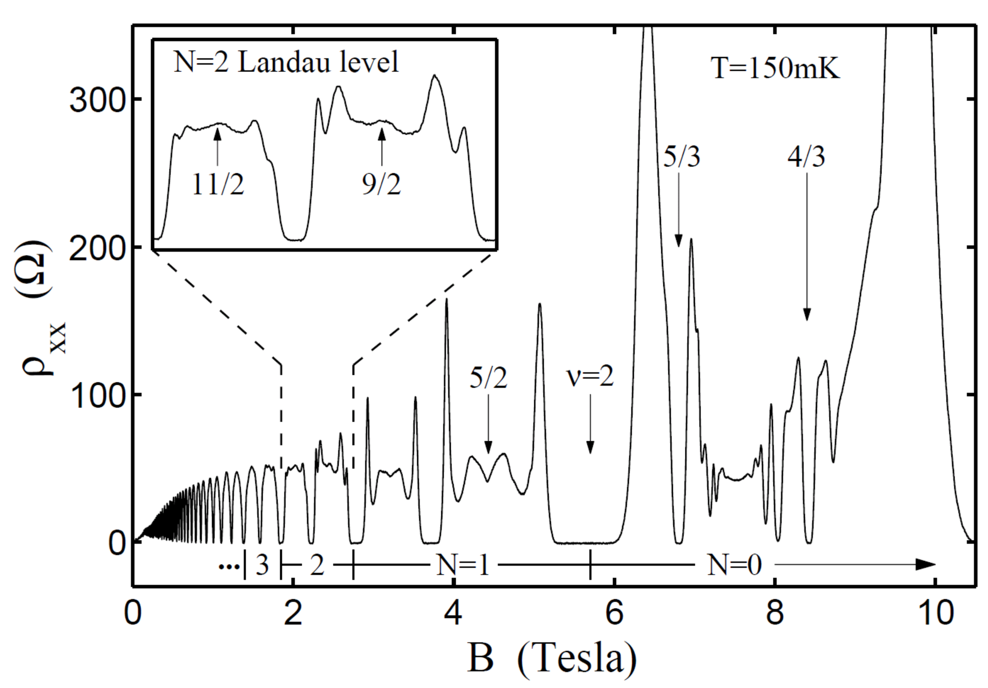
Abstract

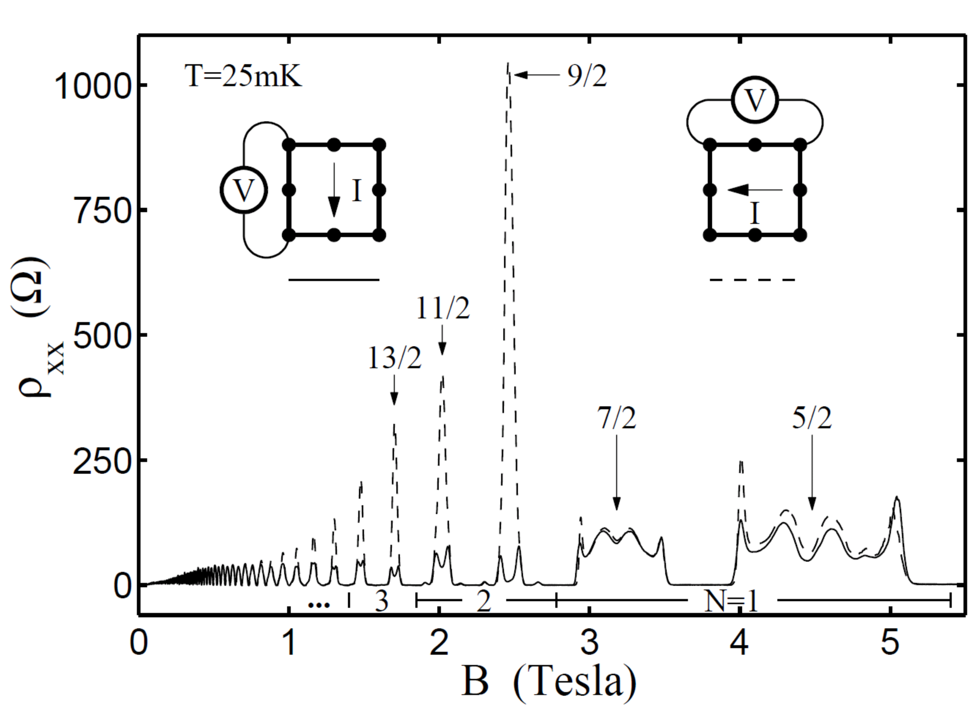
We propose a theory for the orientational pinning of the stripes seen in high Landau levels of GaAs. Rashba and Dresselhaus spin-orbit interactions are the pinning agents. The preferred axis is determined by the relative sign of Rashba and Dresselhaus constants. The pinning energy scale is on the order of *meV* in agreement with experiments.

Stripes in high Landau levels

There is no evidence of fractional quantum Hall states of the Laughlin type for high LL's, $n \geq 2$, i.e. $\nu \geq 4$[1],



Instead, near half filling, highly anisotropic longitudinal transport, and non-quantized Hall transport is seen[1]:



States had been theoretically anticipated to be charge density waves running along a single direction: *Stripes*[2].

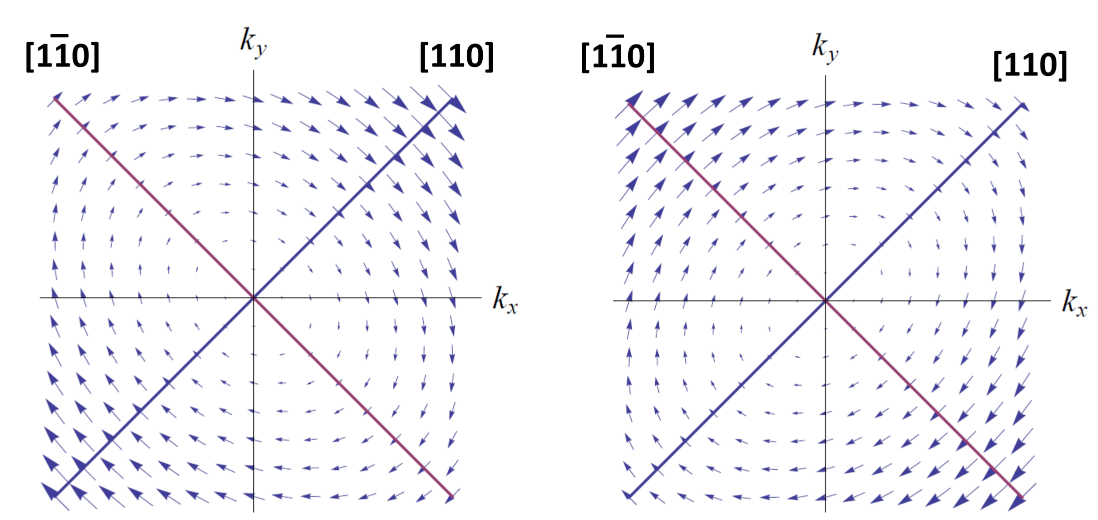
Rashba and Dresselhaus SOC

For the conventional parabolic band model, there is independent orbital and spin rotational symmetry,

$$H_0 = \frac{\pi^2}{2m^*} - \mu \cdot B = \hbar\omega_c(\hat{n}+1/2) + \frac{1}{2}|g^*|\mu_B B\sigma_z$$

Focusing on narrow well limit allows to neglect cubic spin-orbit terms. Rashba and Dresselhaus spin-orbit interactions break rotational symmetry,

$$H_{\text{SO}} = \alpha\left(\sigma_x\pi_y - \sigma_y\pi_x\right) + \beta\left(\sigma_y\pi_y - \sigma_x\pi_x\right)$$



Effective Rashba-Dresselhaus SOC magnetic field in momentum space. Left α=-0.3 β. Right α=0.3 β.

Typical energy scales in GaAs wells (in Kelvin, B in Tesla, $n \sim 3 \times 10^{11}$ cm^{-2}):

Cyclotron	$\hbar\omega_c$	$20.3KB$
Zeeman	$\lvert g^*\rvert\mu_B B$	$0.30KB$
Rashba	$\hbar\alpha/l$	$0.09K\sqrt{B}$
Dresselhaus	$\hbar\beta/l$	$0.28K\sqrt{B}$
Coulomb	$e^2/\epsilon l$	$50.6K\sqrt{B}$

Perturbation theory in α, β is justified:

$$|n,\sigma\rangle = \frac{1}{\sqrt{Z(n,\sigma)}}\left(|n,\sigma\rangle_0 + |n,\sigma\rangle_1 + |n,\sigma\rangle_2 + \cdots\right)$$

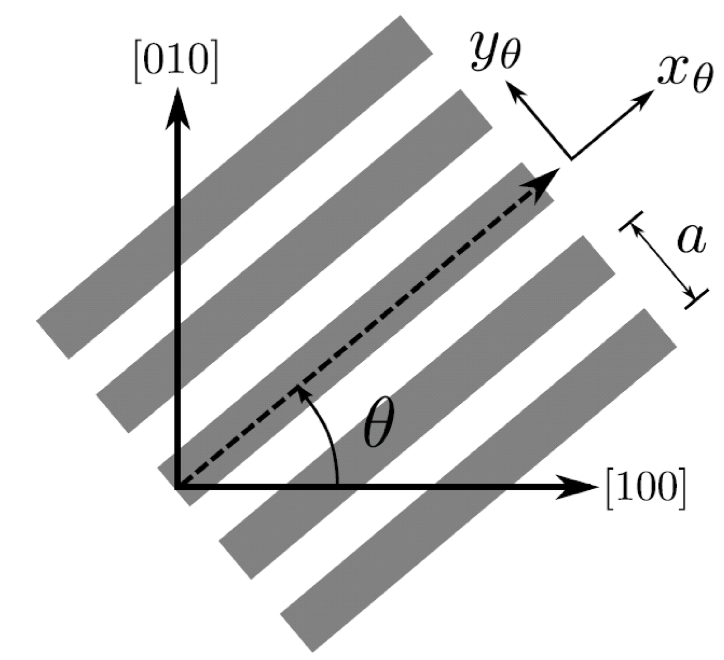
$$|n,\uparrow\rangle_1 = -\frac{i\epsilon_R\sqrt{n}}{1+\epsilon_Z}|n-1,\downarrow\rangle_0 + \frac{\epsilon_D\sqrt{n+1}}{1-\epsilon_Z}|n+1,\downarrow\rangle$$

$$|n,\downarrow\rangle_1 = -\frac{i\epsilon_R\sqrt{n+1}}{1+\epsilon_Z}|n+1,\uparrow\rangle_0 - \frac{\epsilon_D\sqrt{n}}{1-\epsilon_Z}|n-1,\uparrow\rangle_0$$

$$\epsilon_R \equiv \frac{\sqrt{2}\alpha}{l\omega_c}, \quad \epsilon_D \equiv \frac{\sqrt{2}\beta}{l\omega_c}$$

Hartree-Fock pinning energy

Consider a trial Slater determinant describing a rotated stripe state,



The state fills rotated Landau gauge states inside the gray areas of the figure. The orientation, θ ,and the period, *a,* are variational parameters.

$$\Psi_{n,\sigma}(\theta,a) = \prod_{k\in K} C^{\theta\dagger}_{n\sigma k}\Psi^0_{n,\sigma}$$

The influence of spin-orbit is captured by their modification of the density form factors,

$$F_{n,\sigma}(q) \equiv \langle n,\sigma|e^{iq\cdot z\times\pi}|n,\sigma\rangle$$

$$\bar{V} = \frac{1}{A}\sum_{i<j}\sum_{q\neq 0} v_q|F_{n\sigma}(q)|^2 e^{iq(c_i-c_j)}$$

$$c = r - l^2\hat{z}\times\pi/\hbar$$

The leading anisotropic contribution to form factors,

$$F^{\text{ani}}_{n,\uparrow}(q) = \epsilon_R\epsilon_D q_x q_y e^{-|q|^2/4} \times \cdots$$
$$\left(\frac{2L^2_{n-1}\left(|q|^2/2\right)}{1-\epsilon_Z^2} - \frac{L^2_n\left(|q|^2/2\right)}{1-\epsilon_Z} - \frac{L^2_{n-2}\left(|q|^2/2\right)}{1+\epsilon_Z}\right)$$

$$E_{H/F} = \frac{N_\phi}{4\pi l^2}\sum_{n=-\infty}^{\infty} v_{H/F}\left(\frac{2n\pi}{a}\right)\left[\frac{\text{Sin}(n\pi\nu)}{n\pi}\right]^2$$

The main result: The energy of stripes as function of angle θ , is:

$$E^{\text{ani}}(\theta,a) = N_\phi\varepsilon(a)\,\text{Sin}2\theta\left(\epsilon_R\epsilon_D\frac{e^2}{\epsilon l}\right)$$

Results and discussion

(1) The pinning axes are automatically predicted to be [110] or [1$\bar{1}$0], without any fine tuning. These are the pinning directions observed experimentally[1].

(2) The pinning energy scale agrees with in-plane field estimates. This has been estimated to be a few mK[3,4],

$$\epsilon_R\epsilon_D\frac{e^2}{\epsilon l} \sim \frac{6.1mK}{\sqrt{B[T]}}$$

(3) We predict that the orientation of the stripes would switch between [110] and [1$\bar{1}$0] when the relative sign of Rashba and Dresselhaus constants is changed. This could be realized by changing the sign of the effective electric field arising from the structural inversion asymmetry of the well.

Pinning constants and stripe periods at typical half-filled high Landau levels:

ν_{total}	9/2	11/2	13/2	15/2	17/2	19/2
a_0/l	6.19	6.19	7.20	7.20	8.08	8.08
ε	−0.093	−0.093	−0.102	−0.102	−0.113	−0.113
a_0^{RPA}/l	6.51	6.57	7.51	7.54	8.37	8.41
ε^{RPA}	−0.072	−0.075	−0.089	−0.091	−0.102	−0.105

Results are robust to the details of the electron-electron repulsion, e.g., with static screened RPA interactions, pinning energy is only reduced by about 10%.

$$v_q \to v_q^{RPA} = \frac{v_q}{1-v_q\chi_q}$$

Work supported by the Welch foundation grant TBF1473, by the DOE Division of Materials Sciences and Engineering grant DE-FG03-02ER45958, and by the NRI SWAN program.

References

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June 2013