# Universal behavior of the magnon gaps in doped quasi-2D antiferromagnets



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## Abstract

Within the framework of the anisotropic quantum non-linear  $\sigma$ -model (QNL $\sigma$ M) we calculate doping and temperature dependence of the magnon gaps in a prototypical quasi-2D antiferromagnet  $La_{2-x}Sr_xCuO_4$  (x < 0.02) and obtain good agreement with experiments. It is shown that the reduction of the magnon gaps relative to their x = 0 value weakly depends on the anisotropies of the parent compound. Since the DM gap is highly sensitive to rare-earth element doping, this prediction could be tested on  $La_{2-x-y}Eu_ySr_xCuO_4$  (LESCO) and  $La_{2-x-y}Nd_ySr_xCuO_4$  (LNSCO).

#### Motivation

In lightly doped  $La_{2-x}Sr_xCuO_4$  (LSCO):

- Dzyaloshinskii-Moriya (DM) and XY anisotropies generate small magnon gaps ( $\sim 2\%$  of the AF exchange constant) [1].
- The gaps decrease with doping much faster than is expected from reduction of the underlying anisotropies [2, 3].

#### Questions:

- What is the mechanism responsible?
- Is the magnon gap dependence on doping sensitive to the details of the parent compound?

## The model

Anisotropic non-linear  $\sigma$ -model (NL $\sigma$ M) coupled to the dipole fields representing holes:

$$\mathcal{L}_{s} = \frac{\rho_{s}}{2c} [(\partial_{0}\mathbf{n})^{2} + (\partial_{i}\mathbf{n})^{2} + (\frac{\omega_{0}^{\alpha}}{c})^{2} (n^{\alpha})^{2}], \tag{1}$$

$$\mathcal{L}_{s-\text{dipole}} = \frac{g_{d}}{2} \mathbf{P}_{i} \cdot (\mathbf{n} \times \partial_{i}\mathbf{n}), \tag{2}$$

$$\mathcal{L}_{\text{s-dipole}} = \frac{g_{\text{d}}}{c} \mathbf{P}_i \cdot (\mathbf{n} \times \partial_i \mathbf{n}), \tag{2}$$

$$\mathcal{L}_{\text{dipole}} = \frac{1}{2\kappa c} \mathbf{P}_i G_{\mathbf{d}}^{-1} \mathbf{P}_i. \tag{3}$$

 $\mathbf{n}^2 = 1 - \text{staggered magnetization}, \ G_{\mathrm{d}}(\mathrm{i}\omega_n, \mathbf{q}) = \frac{Dq^2}{Dq^2 + |\omega_n|} - \text{dipole field propagator}, \ \omega_0^{\alpha} - \text{bare}$ magnon gaps,  $\rho_s$  – spin stiffness, c – spin-wave velocity,  $g_{\rm d}$  – spin-dipole coupling constant.

## Methods

• Dipole fields can be integrated out  $\Rightarrow$  effective spin interaction term:

$$\mathcal{L}_{\text{int}} = -\frac{g_{\text{d}}^2 \kappa}{4c} n^k \stackrel{\leftrightarrow}{\partial_i} n^l G_{\text{d}} n^k \stackrel{\leftrightarrow}{\partial_i} n^l \stackrel{D \gg 1}{\longrightarrow} -[g_{\text{d}}^2 \kappa/(2c)] \cdot (\partial_i \mathbf{n})^2. \tag{4}$$

- In the diffusive limit  $(D \to \infty)$  one gets pure spin stiffness and spin-wave velocity renormalization ( $\rho_{\rm s} \to \rho_{\rm s} - g_{\rm d}^2 \kappa$ ,  $\rho_{\rm s}/c^2 = {\rm const.}$ ).
- We derive the **effective action** for the QNL $\sigma$ M in the diffusive limit ( $D \gg 1$ ) to the one-loop order:

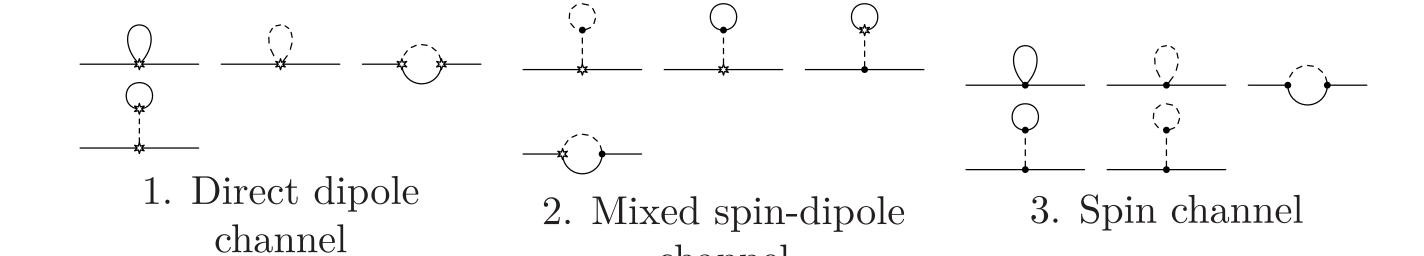
$$S_{\text{eff}} \approx \sum_{\gamma} \frac{\rho_{\text{R}}^{\gamma}}{2c} \int_{0}^{c/T} dx_0 \int d^2x [(\partial_{\mu} n_0^{\gamma})^2 + (\frac{\omega_{\text{R}}^{\gamma}}{c})^2 (n_0^{\gamma})^2],$$
 (5)

where

$$\rho_{\rm R}^{\alpha} = \rho_{\rm s} \left[1 - \frac{c}{\rho_{\rm s}} \sum_{\gamma} \int d\tilde{\mathbf{q}} G_{0\pi}^{\gamma}(\tilde{\mathbf{q}}) + \frac{c}{\rho_{\rm s}} \int d\tilde{\mathbf{q}} G_{0\pi}^{\alpha}(\tilde{\mathbf{q}})\right],\tag{6}$$

$$\omega_{\mathbf{R}}^{\alpha} = \omega_0^{\alpha} \left[ 1 - \frac{c}{\rho_{\mathbf{S}}} \int d\tilde{\mathbf{q}} G_{0\pi}^{\alpha}(\tilde{\mathbf{q}}) \right]. \tag{7}$$

• Alternative approach – linear  $\sigma$ -model (can be easily used away from the diffusive limit). The results obtained within  $NL\sigma M$  are confirmed.

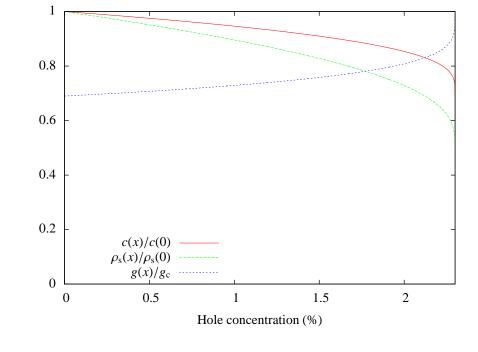


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#### Results

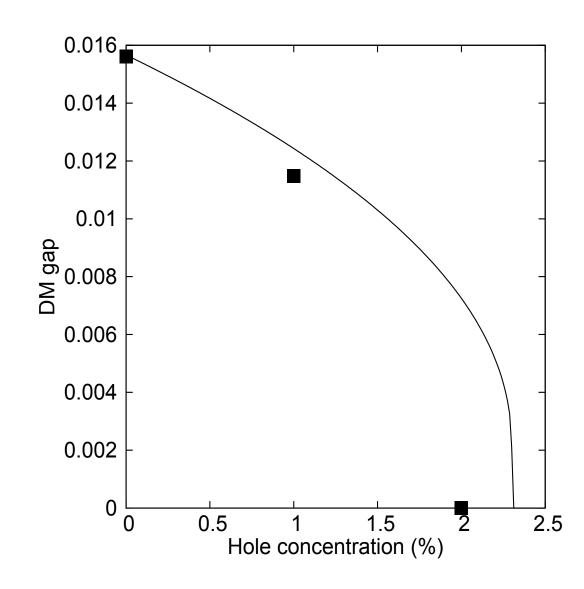
• The dependence of  $\rho_s$  and c on doping is obtained from the empirical scaling law for staggered magnetization (cf. Ref. [4]):

$$\begin{cases} \frac{M^{\dagger}(x)}{M^{\dagger}(0)} \sim (1 - \frac{x}{x_{c}})^{\mu}, \text{ where } x_{c} = 0.023, \mu = 0.236\\ (M^{\dagger})^{2} \sim 1 - \frac{g(x)}{g_{c}} \end{cases}$$
(8)



Doping dependence of the normalized spin stiffness, spin-wave velocity and the  $QNL\sigma M$ coupling constant obtained from Eq. (8).

• Doping and temperature dependence of the magnon gaps in LSCO:



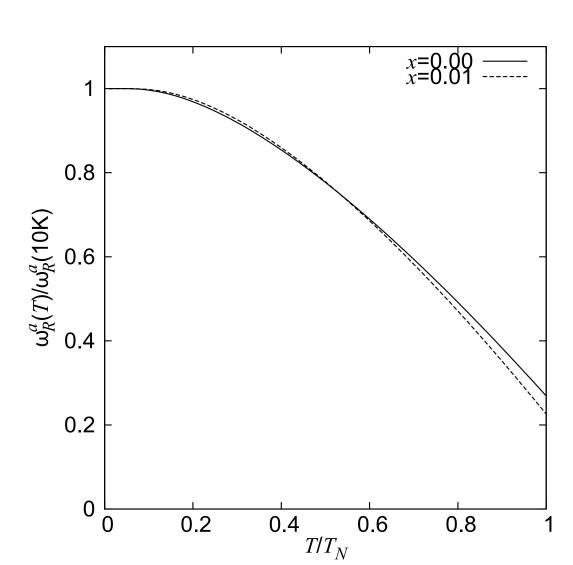


Fig. 2. DM gap vs doping for LSCO (in the units of J). The solid line is the theoretical curve for 10K. Solid squares are the experimental data of Ref. [2]

Normalized DM gap  $\omega_{\rm R}^a$  vs temperature for LSCO (x = 0 and x = 0.01). The solid and dashed lines are the theoretical curves. The Néel temperature  $T_{\rm N}(x)$  is taken from Ref. [4].

• Universal doping dependence of the magnon gaps is predicted for various La-based cuprates:

	$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$				$La_{1.65-x}Nd_{0.35}Sr_xCuO_4$		
,	$\overline{x}$	experiment	this work		$\overline{x}$	experiment	this work
,	0.00	$2.1 \mathrm{meV}$	2.1  meV		0.00	4.5  meV	4.5  meV
	0.01	1.55  meV	1.66  meV		0.01	?	3.68  meV
21% reduction predicted				18% reduction predicted			

**Table 1.** Comparison of the in-plane gaps in LSCO and LNSCO with  $NL\sigma M$ predictions for different Sr doping levels. The experimental values are taken from Refs. [2, 4, 5].

## Summary

- The dependence of the magnon gaps on doping has been discussed within the framework of the non-linear  $\sigma$ -model.
- Good agreement with available experimental data is obtained.
- The relative magnon gap reduction with doping is predicted to be insensitive to the anisotropies of the parent compound.

## References

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