Fractal Dynamics in Chaotic Quantum Transport

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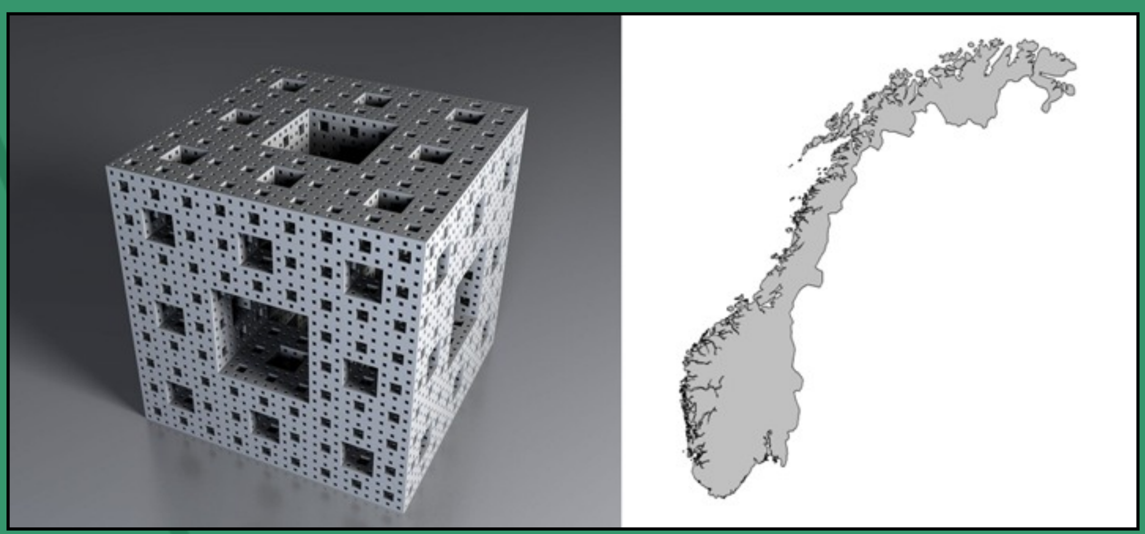
Abstract

- We have carried out quantum transport calculations in real space and real time for a two-dimensional stadium cavity that shows fractal fluctuations in magnetoconductance [1].
- Fractal nature of the magnetoconductance is analyzed with methods that originate from different fields of physics: the variation method [2] and the detrended fluctuation analysis (DFA) [3].

Introduction

- Fractal patterns can be commonly observed in nature, e.g., in snowflakes, fern leaves, coastlines, and also in the fluctuations of human-generated time series as in heartbeats [4] and music [5,6]. These self-affine structures have also been found in magnetoconductance of chaotic, e.g. stadium-shaped quantum wells [7].



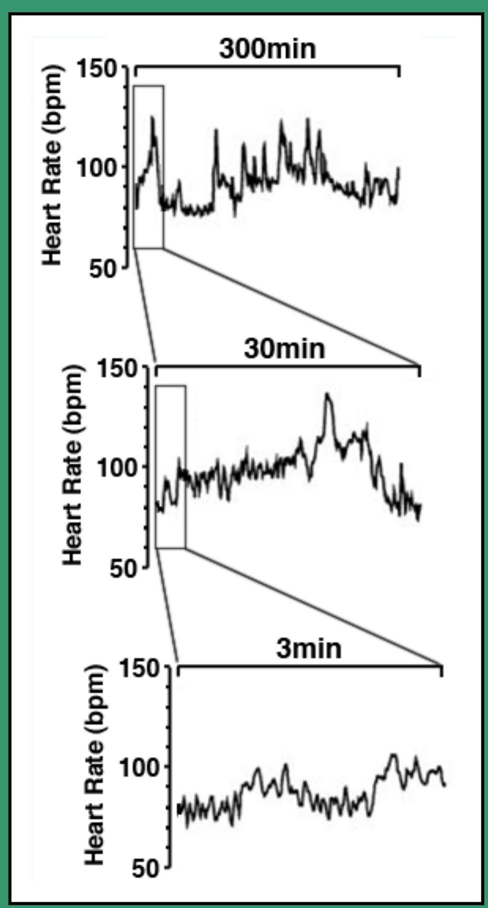


FIG. 1. Left panel: Two examples of spatial self-similarity where the shape looks like the same in all scales. Menger-Sponge curve (3D rendering from Wikipedia) which is made by carving out down-scaled cubes from a larger cube in repeating fashion, and the coastline of Norway where the amount of visible detail is scale-independent. Right panel: Human heart rate data from Goldberger et al. [4]. The temporal data has similar statistical properties on all time scales.

Experimental setup

- We consider a gate-defined stadium-shaped quantum well formed in the 2D electron gas in the AlGaAs/GaAs heterojunction.
- Measured magnetoconductance shows fractal fluctuations over two orders of magnitude in magnetic field.

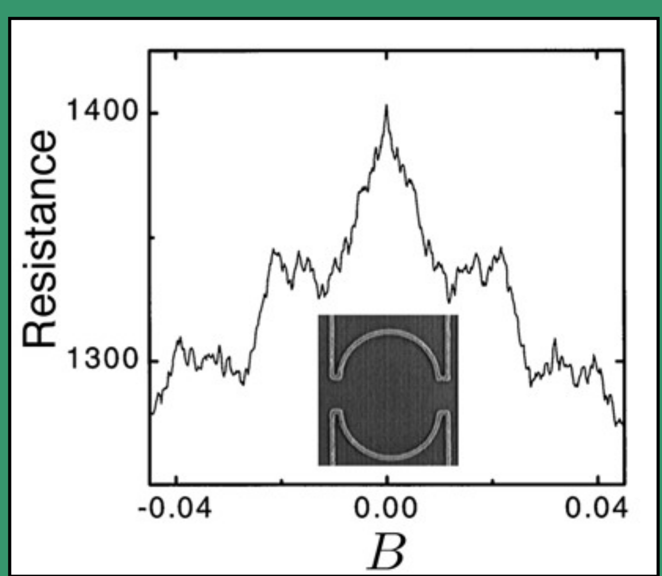


FIG. 2. Resistance of a 2DEG stadium quantum well as a function of magnetic field from Sachrajda et al. [7]. Inset: A scanning electron microscope image of the device.

Methods

Transport scheme

- Time-dependent single-particle Schrödinger equation is solved for a wave packet in an external potential that models the experimental setup. The procedure is repeated for a large ensemble of static, perpendicular magnetic fields. The Hamiltonian of the system in atomic units is:

$$\hat{H} = \frac{1}{2}[-i\nabla + \mathbf{A}(\mathbf{r})]^2 + V_{\rm ext}(\mathbf{r})$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential in linear gauge.

- Transmission coefficient (comparable to conductance) is calculated by integrating the electron density at the output lead as a function of time.
- Time evolution operator is approximated with the fourth-order Taylor expansion and all simulations are performed with the `Octopus` code [8].

Fractal analysis

- Fractal dimension quantifies how the pattern scales differently than the space it is embedded in, i.e., how "dense" the pattern is.
- Idea of *variation analysis* is to build a cover for the curve from many sets of size L and see how the total area of the cover scales as a function of L. The scaling exponent of this relation is the fractal dimension [2].
- *Detrended fluctuation analysis* is a method that was first used to analyze the organization of DNA nucleotides [3] but is nowadays more common in the field of time-series analysis.

The standard procedure of DFA consists of the following four steps:

1. Calculate the cumulative sum of the series' deviation from the mean.
2. Divide integrated series into small windows.
3. Fit a polynomial of nth degree that represents the trend in the window.
4. Calculate standard deviation with respect to the local trend and average all results. The standard deviation obtained this way is a power law function of the window size. The scaling exponent α is used to classify the nature of correlations in the time series.

The fractal dimension can be calculated with a slightly modified version of DFA.

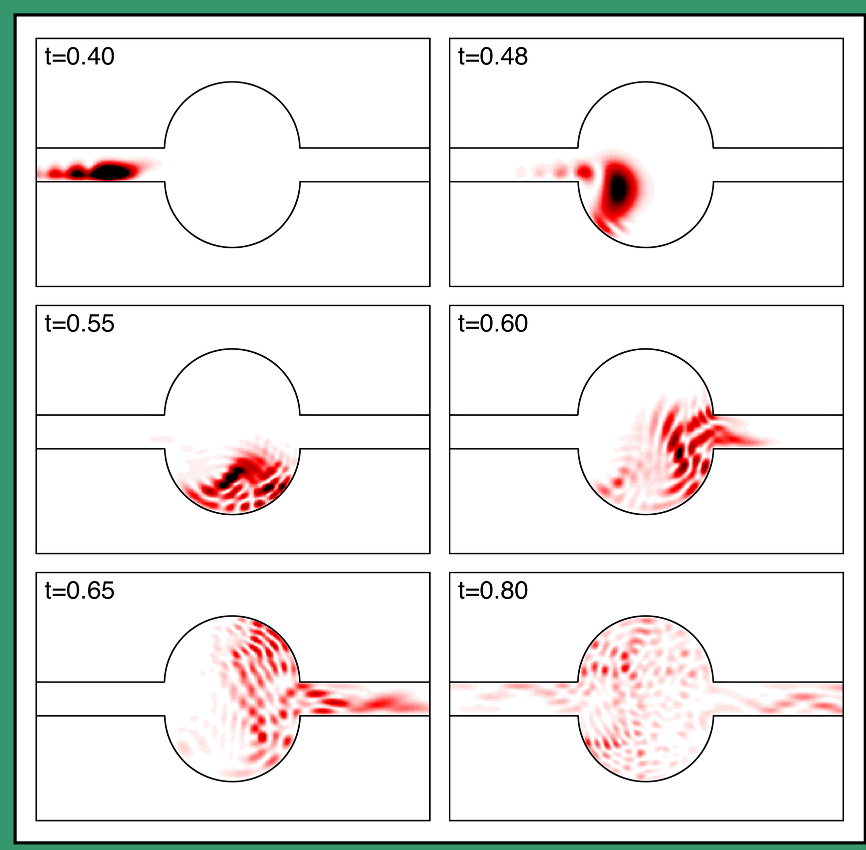


FIG. 3. Snapshots of the electron density during a transport simulation with the magnetic flux $\Phi/\Phi_0 = 20$. The input and output leads extend further to the left and right.

Results

- Quantum mechanical transmission coefficient displays complex dependence on magnetic flux through the stadium quantum well.
- Fractal behavior of the magnetoconductance can be seen over two orders of magnitude.
- Calculated fractal dimension D=1.3 is in excellent qualitative agreement with the experimental result D=1.25 [7].
- DFA is a well suited method for fractal analysis of chaotic quantum transport.

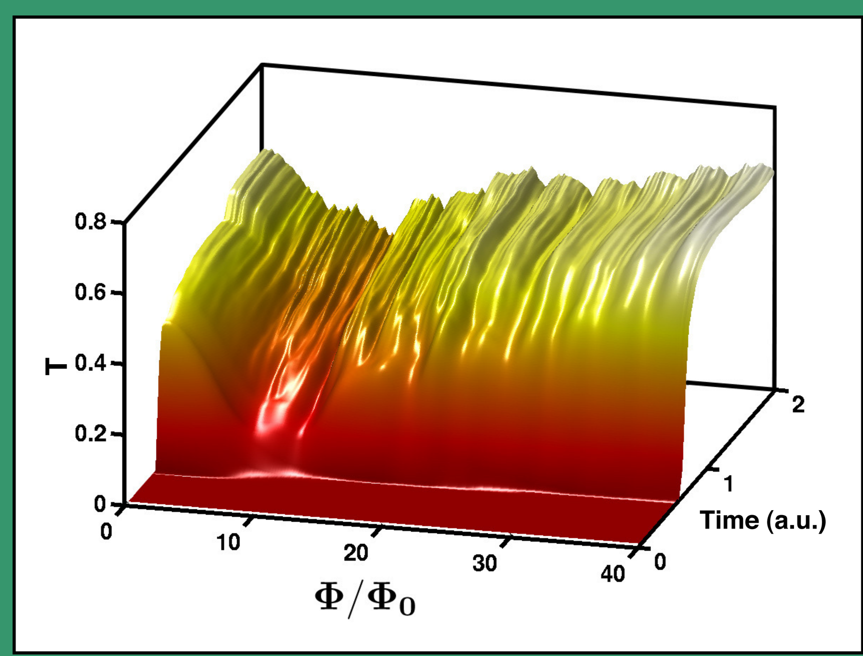


FIG. 4. Transmission coefficient as a function of time and magnetic flux through the stadium quantum well.

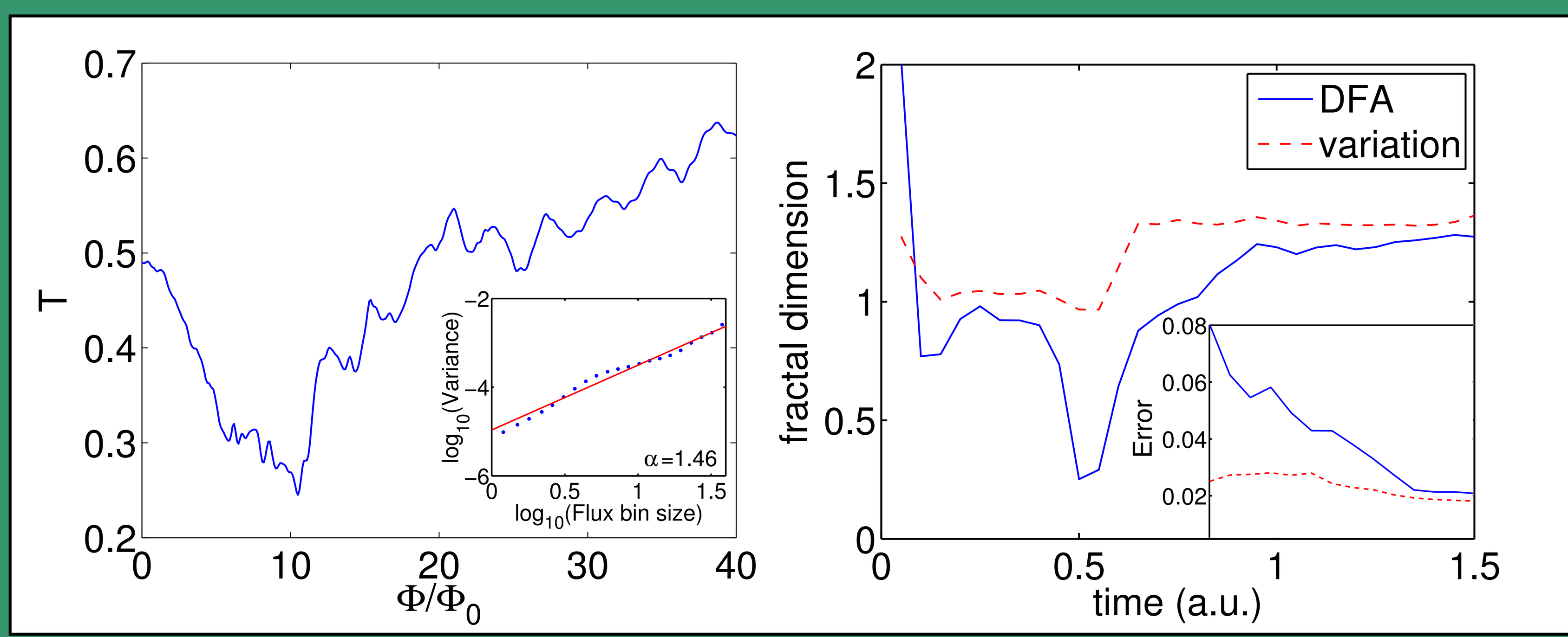


FIG. 5. Left panel: Cross section of the transmission coefficient from FIG. 4 at t=1.4 displays oscillations in several size scales as a function of magnetic flux. Inset: DFA linear fit showing fractal behavior over two orders of magnitude. Right panel: Fractal dimension of the conductance oscillations as a function of time. Inset: Error of the linear fit used in obtaining fractal dimension.

Outlook

- It was recently shown in Ref. [9] that the disorder can have a large effect on the magnetoconductance fingerprint of the system. The effect of impurities could be examined by adding scatterers to the model potential.
- Current simulations consist of single-electron transmission coefficient. The effect of electron-electron interactions could also be examined by using time-dependent density-functional theory.

Acknowledgements

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